Introduction to Quantum Information- III

General Ideas

The Goal: Define and Calculate the following:

Classical Channel

Quantum Channel

One Capacity

Four different capacities

یک کد گذاری ساده برای تصحیح خطا

x $P(y \mid x)$ y

$$0 \longrightarrow 000$$

$$1 \longrightarrow 111$$

نرخ مخابره $\quad R = \frac{1}{3}$

This is a very simple letter code.

Block Codes

01001001001000100100111101001010010 1001

k

0100100100010110110110001001001111010001101011011101001 01001

n

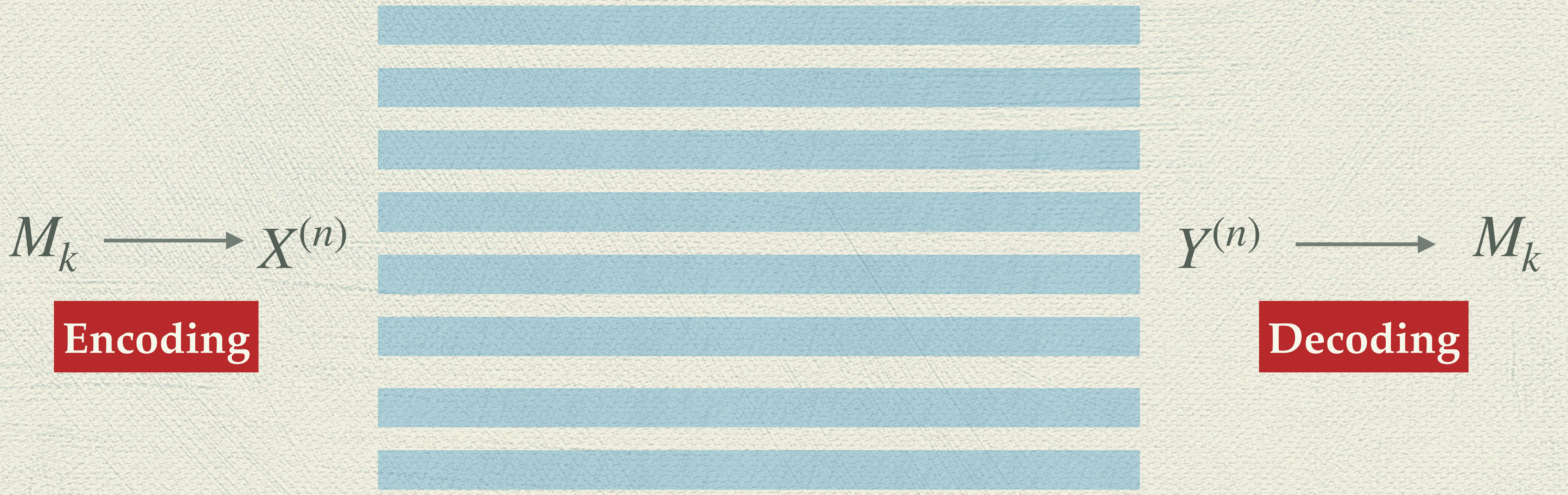
$$M_k \longrightarrow X^{(n)} \qquad Y^{(n)} \longrightarrow M_k$$

Encoding **Decoding**

$$R(X) = \lim_{n \longrightarrow \infty} \frac{k}{n}$$

$$P_{error} \longrightarrow 0$$

تعریف: ظرفیت کانال کلاسیک



$$C = \underset{X}{Max}\ R(X)$$

می بایست روی تمام منبع ها بیشینه نرخ را حساب کرد.

محاسبه: ظرفیت کانال کلاسیک

$$x \quad P(y|x) \quad y$$

$$I(X:Y) = H(X) + H(Y) - H(X,Y)$$

$$C = \underset{P(x)}{Max}\ I(X:Y)$$

محاسبه: ظرفیت کانال کلاسیک

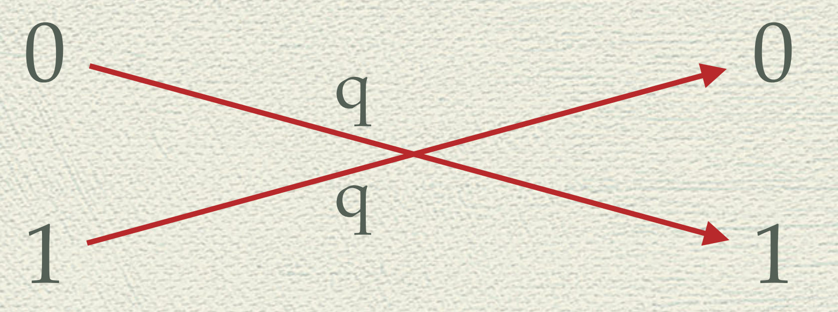
x $P(y|x)$ y

$$P(x,y) = P(y|x)P(x)$$

$$H(X) = -\sum_{x} P(x)\log P(x) \qquad H(Y) = -\sum_{y} P(y)\log P(y)$$

$$H(X,Y) = -\sum_{x,y} P(x,y)\log P(x,y)$$

مثال: محاسبه ظرفیت یک کانال متقارن



$$I(X:Y) = q\log q + (1-q)\log(1-q) + \left[H(\lambda)\right]$$

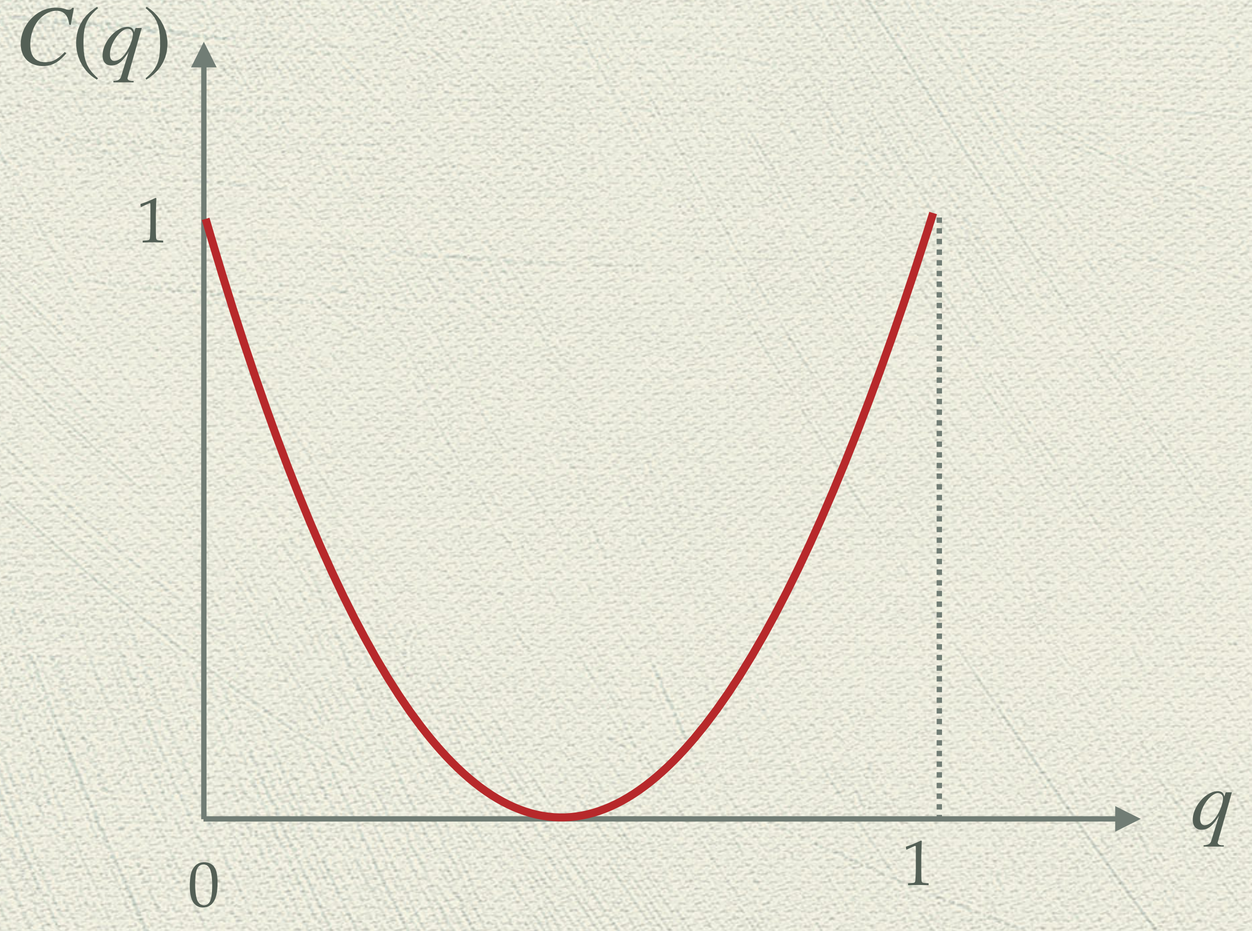
$$\lambda = p + q - 2pq$$

بیشینه کردن

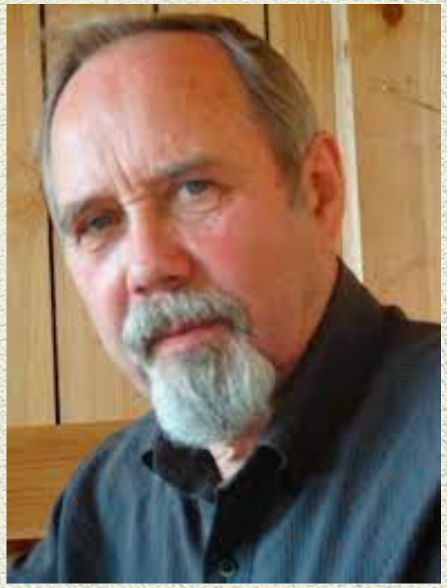
$$p + q - 2pq = \frac{1}{2} \longrightarrow p = \frac{1}{2}$$

نتیجه نهایی محاسبه

$$C(q) = 1 + q\log q + (1-q)\log(1-q)$$



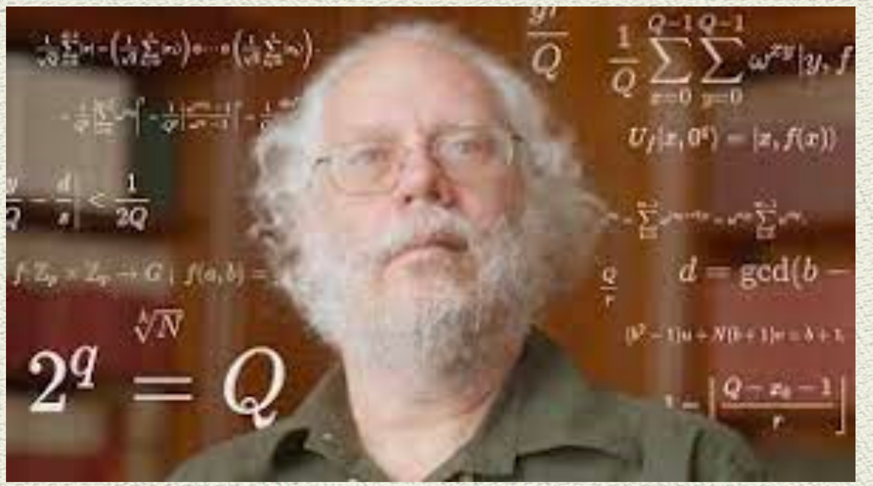
ظرفیت های کانال های کوانتومی



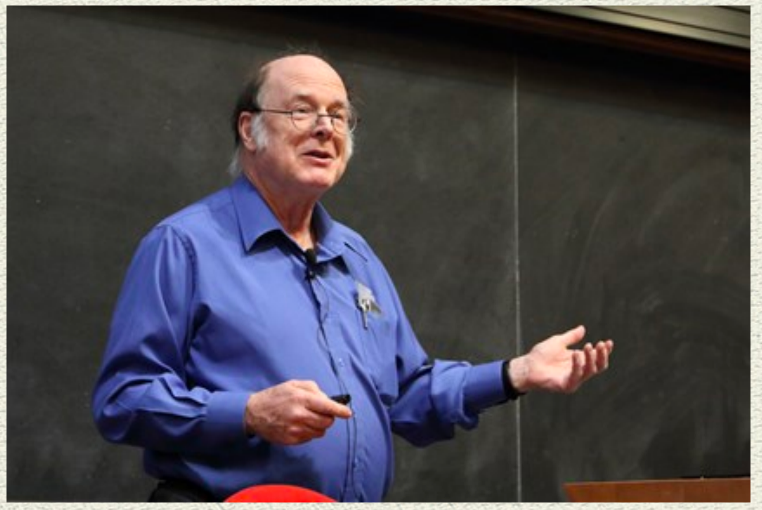
Holevo

Shumacher

Westmoreland



Peter Shor



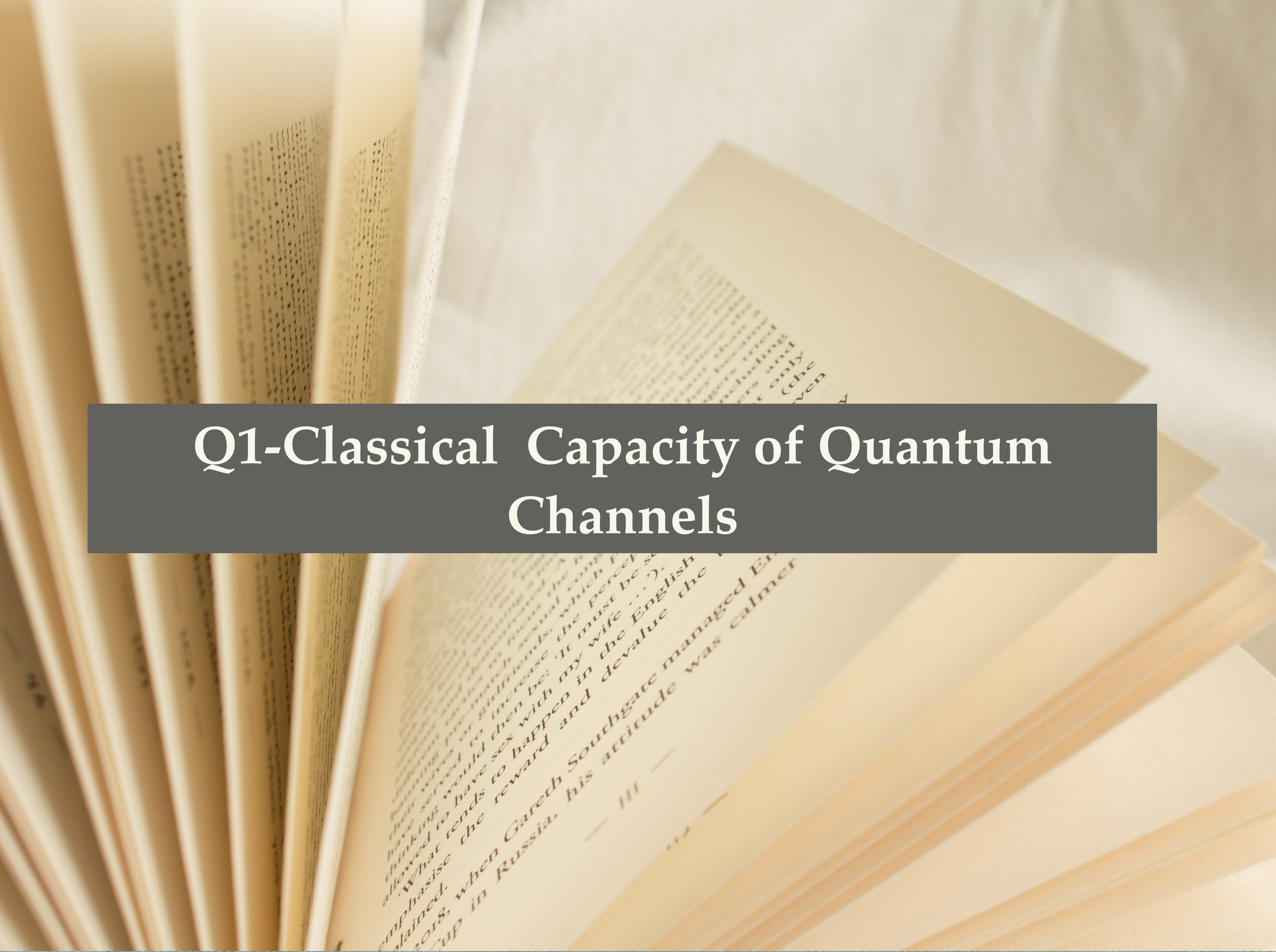
Charles Bennett

یک- برای یک کانال کوانتومی انواعی از ظرفیت تعریف می شود.

دو- تعریف این ظرفیت ها یک چیز است و تقلیل
این تعریف ها به یک رابطه بسته یک چیز دیگر.

سه -حتی محاسبه آن رابطه بسته هم آسان نیست
(به دلیل بیشینه کردن روی یک فضای پیوسته)

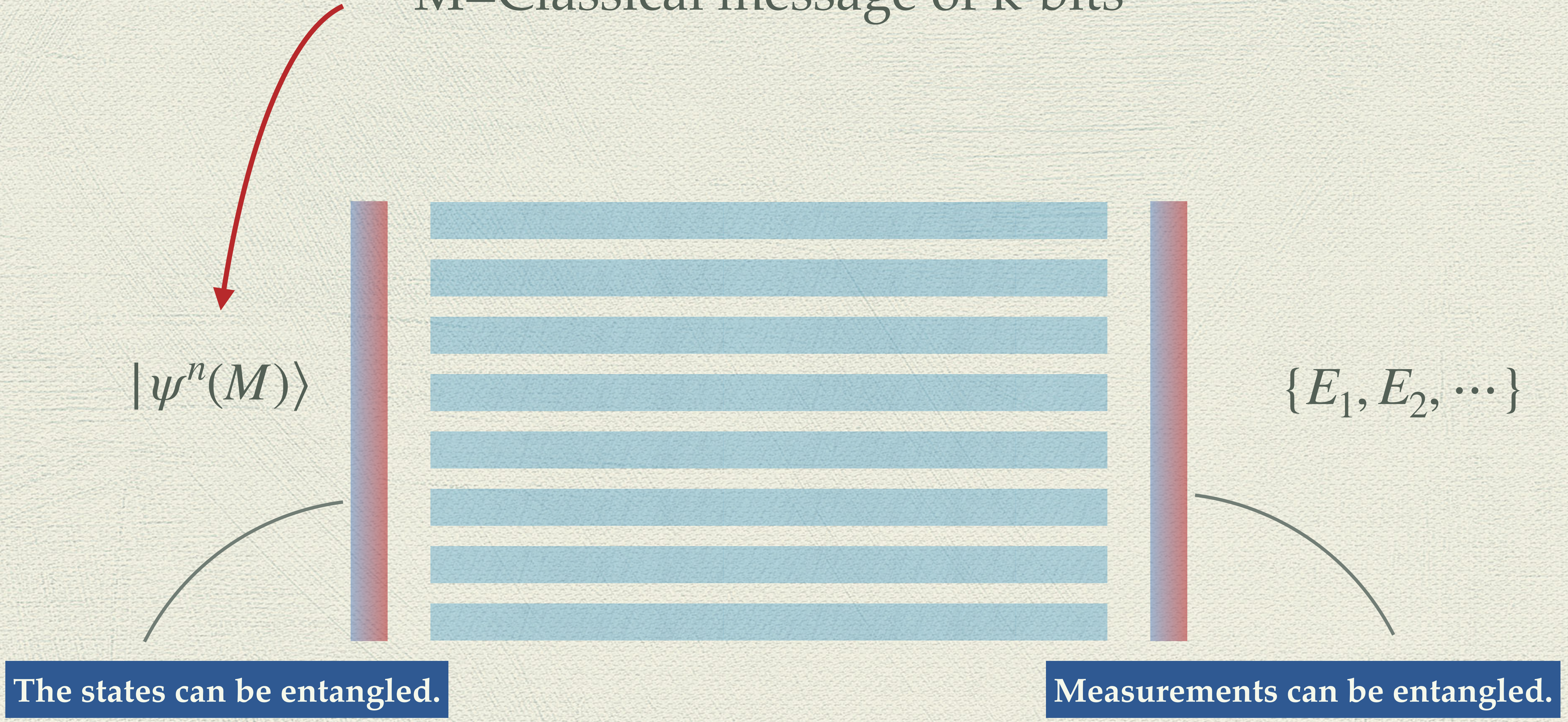
چهار- برخلاف ظرفیت کلاسیک این ظرفیت ها عموما جمع پذیر نیستند (به دلیل درهم تنیدگی).



Q1-Classical Capacity of Quantum Channels

Definition: Classical Capacity of Quantum Channels

M=Classical message of k-bits



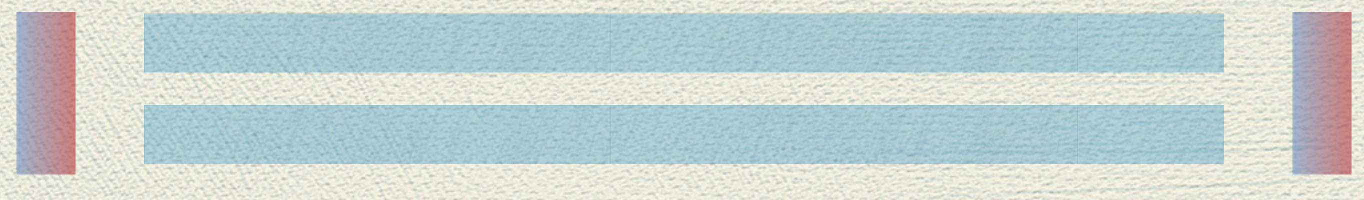
$|\psi^n(M)\rangle$ =Quantum state of n-qubits

این حالت ها بر هم عمودند.

این حالت ها ممکن است برهم عمود نباشند
و عموما هم آمیخته هستند.

Example:

$\rho_M^{(n)}$



$\{\,|\phi^+\rangle, |\psi^+\rangle, \cdots\}$

$00 \longrightarrow |\phi^+\rangle$

$01 \longrightarrow |\psi^+\rangle$

$10 \longrightarrow |\psi^-\rangle$

$11 \longrightarrow |\phi^-\rangle$

Quantum Channel →

ρ_{00}

ρ_{01}

ρ_{10}

ρ_{11}

$$P_{error}(00) = 1 - \langle \phi^+ | \rho_{00} | \phi^+ \rangle$$

$$P_{error}(01) = 1 - \langle \psi^+ | \rho_{01} | \psi^+ \rangle$$

$$\dots\dots\dots\dots$$

$$\dots\dots\dots\dots$$

$$P_{error} = \frac{P_{error}(00) + P_{error}(01) + \cdots}{4}$$

$\rho_M^{(n)}$

$\{E_M, \cdots\}$

$$P_{error}(M) = 1 - Tr\left(E_M \Phi^{\otimes n}(\rho_M^{(n)}) \right)$$

$$\overline{P}_{error} = \frac{\sum_M 1 - Tr\left(E_M \Phi^{\otimes n}(\rho_M^{(n)}) \right)}{|M|}$$

Achievable rate for the source X

$$R(X) = \lim_{n \longrightarrow \infty} \frac{\log |M|}{n} = \lim_{n \longrightarrow \infty} \frac{k}{n}$$

when $P_{error} \rightarrow 0$

تعریف ظرفیت کلاسیک یک کانال کوانتومی

$$C := \underset{X}{Max}\; R(X)$$

ماکزیمم گیری روی تمام منبع ها صورت می گیرد.

این همان تعریف کلی و دقیق ظرفیت است و هنوز فرمول بسته نیست.

فرمول بسته برای ظرفیت کلاسیک کانال کوانتومی؟

فقط در یک حالت وجود دارد.

$C^{(1)}$

$\rho_1 \otimes \rho_2 \otimes \cdots \rho_n$

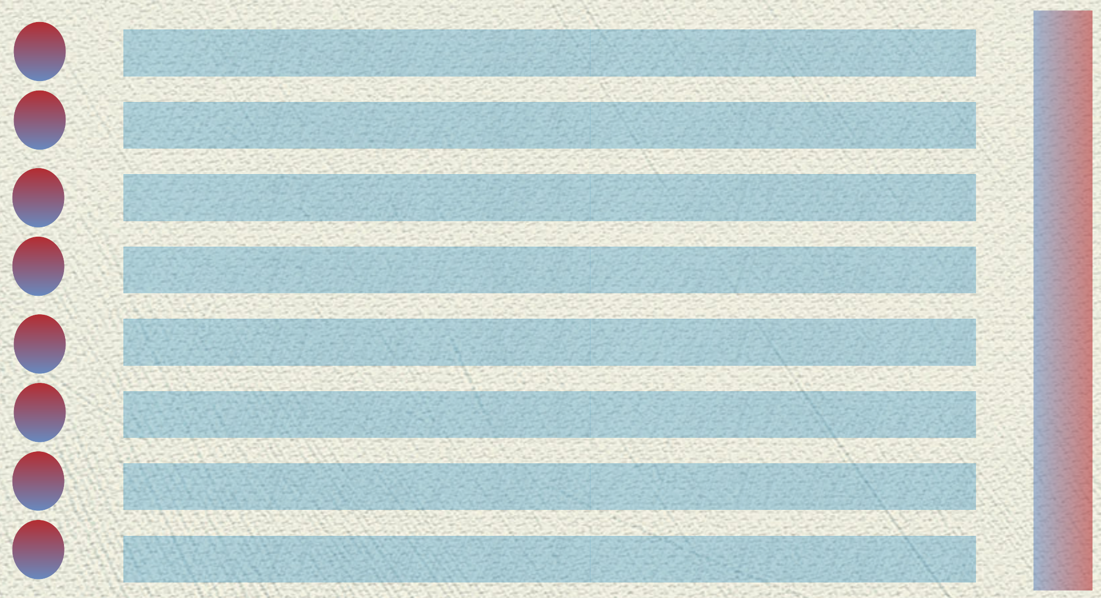
$\{E_M, \cdots\}$

وقتی حالت های ورودی به صورت ضربی باشند: $\rho_1 \otimes \rho_2 \otimes \cdots \rho_n$

فرم بسته برای ظرفیت کلاسیک کانال کوانتومی در یک حالت خاص: $C^{(1)}$

Holevo-Schumacher-Westmoreland

$\rho_1 \otimes \rho_2 \otimes \cdots \rho_M$



$\{E_M, \cdots\}$

Holevo-Schumacher-Westmoreland
Or
Product State Capacity

$$C^{(1)} = \chi^*(\Phi)$$

$$\chi^*(\Phi) = \underset{\{p_i\ ,\ \rho_i\}}{Max}\ \chi(\{p_i, \rho_i\})$$

$$\chi(\Phi) = S\Big[\sum_i p_i \Phi(\rho_i)\Big] - \sum_i p_i S(\Phi(\rho_i))$$

Example 1: $C^{(1)}$ Capacity of Depolarizing Channel

$$\Phi(\rho) = (1-p)\rho + \frac{p}{2}I$$

We take an ensemble $\{p_i, \rho_i\}$ and calculate

$$\chi(\Phi) = S\Big[\sum_i p_i \Phi(\rho_i)\Big] - \sum_i p_i S(\Phi(\rho_i))$$

$$\sum_i p_i \Phi(\rho_i) = \Phi(\sum_i p_i \rho_i) = \Phi(\rho) \qquad \Phi(\rho_i) = (1-p)\rho_i + \frac{p}{2}I$$

$$\chi(\{p_i, \rho_i\}) = S\Big[(1-p)\rho + p\frac{I}{2}\Big] - \sum_i p_i S\Big[(1-p)\rho_i + \frac{p}{2}I\Big]$$

The input ensemble can be taken to be pure, Thm. 13.3.2. Mark Wilde (arxiv.org.1106.1445.).

To maximize the first term, we take $\rho = \frac{I}{2}$.

For the second term:

Since $S(U\sigma U^\dagger) = S(\sigma) \quad \forall\ U, \sigma$

$$S[(1-p)\,|\psi_i\rangle\langle\psi_i\,| + p\frac{I}{2}] = S[(1-p\,|+\rangle\langle+| + p\frac{I}{2}]$$

Therefore it is important to know the minimum output entropy states.

$$\chi(\{p_i, \rho_i\}) = S\Big[(1-p)\rho + p\frac{I}{2}\Big] - \sum_i p_i S\Big[(1-p)\rho_i + \frac{p}{2}I\Big]$$

$$= 1 - S\Big[(1-p)\,|+\rangle\langle+| + \frac{p}{2}I\Big]$$

$$C^{(1)}(p) = 1 + \frac{p}{2}\log\frac{p}{2} + (1-\frac{p}{2})\log(1-\frac{p}{2})$$

$$C^{(1)}(0) = 1 \qquad C^{(1)}(1) = 0$$

Example 2: $C^{(1)}$ Capacity of Bit-flip Channel

$$\Phi(\rho) = (1-p)\rho + pX\rho X$$

We take $\rho = \frac{I}{2}$ to maximize the first term.

$$\chi(\{p_i, \rho_i\}) = S\big[(1-p)\rho + pX\rho X\big] - \sum_i p_i S\big[(1-p)\rho_i + pX\rho_i X\big]$$

Intuition: $\{p_i, |\psi_i\rangle\} = \{1/2, |+\rangle; 1/2, |-\rangle\}$

Invariant States

$$\chi(\{p_i, \rho_i\}) = S\big[(1-p)\rho + pX\rho X\big] - \sum_i p_i S\big[(1-p)\rho_i + pX\rho_i X\big]$$

Intuition: $\{p_i, |\psi_i\rangle\} = \{1/2, |+\rangle; 1/2, |-\rangle\}$

$$\chi = S(\frac{I}{2}) - \frac{1}{2}S(|+\rangle\langle+|) - \frac{1}{2}S(|-\rangle\langle-|) = 1$$

C=1 as it should be.

Example 3: $C^{(1)}$ Capacity of Amlitude-Damping Channel

$$\Phi(\rho) = A_0 \rho A_0^\dagger + A_1 \rho A_1^\dagger$$

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}$$

Intuition: $\{p_i, |\psi_i\rangle\} = \{p, |0\rangle;\ 1-p, |\psi\rangle\}$

Intuition: $\{p_i, |\psi_i\rangle\} = \{p, |0\rangle;\ 1-p, |\psi\rangle\}$

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad |\psi\rangle\langle\psi| = \begin{pmatrix} |a|^2 & a\bar{b} \\ \bar{a}b & |b|^2 \end{pmatrix}$$

$$\rho = \begin{pmatrix} p + (1-p)|a|^2 & (1-p)a\bar{b} \\ (1-p)\bar{a}b & (1-p)|b|^2 \end{pmatrix}$$

$$\Phi(|0\rangle\langle 0|) = |0\rangle\langle 0|$$

$$\Phi(|\psi\rangle\langle\psi|) = \begin{pmatrix} |a|^2 + \gamma|b|^2 & a\bar{b} \\ \overline{a}b & (1-\gamma)|b|^2 \end{pmatrix}$$

$$\Phi(\rho) = \begin{pmatrix} p + (1-p)|a|^2 + \gamma(1-p)|b|^2 & (1-p)a\bar{b} \\ (1-p)\overline{a}b & (1-\gamma)(1-p)|b|^2 \end{pmatrix}$$

$$\chi_\gamma(p, a) = S[\Phi(\rho)] - (1-p)S\big[\Phi(|\psi\rangle\langle\psi|)\big]$$

$$\chi(p,a) = S[\Phi(\rho)] - (1-p)S\big[\Phi(|\psi\rangle\langle\psi|)\big]$$

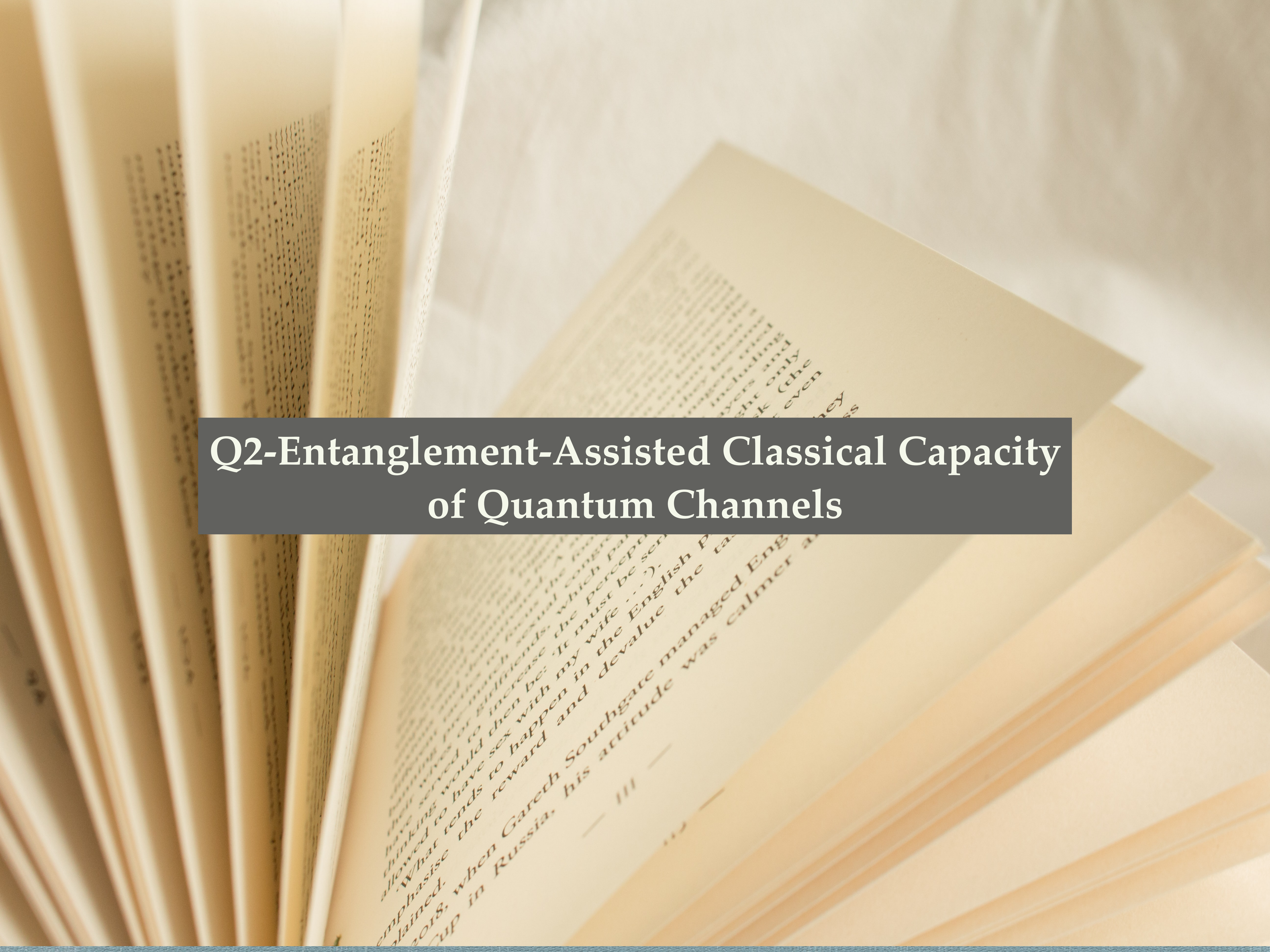
Using $S(\sigma) = S(U\sigma U^{\dagger})$ we can take a and b to be real.

$$a = \cos\theta \qquad\qquad b = \sin\theta$$

$$\chi(p,\theta) = S[\Phi(\rho)] - (1-p)S\big[\Phi(|\psi\rangle\langle\psi|)\big]$$

χ^* and hence C can be calculated only numerically.

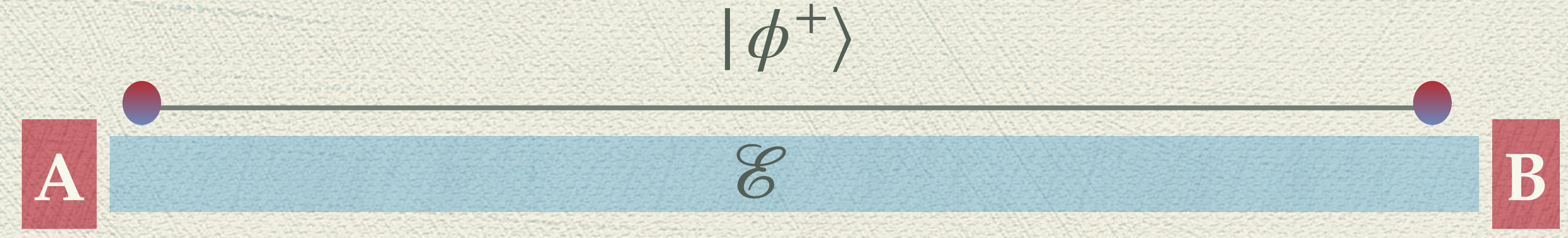
Q2-Entanglement-Assisted Classical Capacity of Quantum Channels



Q2: Entanglement-Assisted Classical Capacity of Quantum Channels

بین فرستنده و گیرنده به دلخواه حالت های درهم تنیده ماکزیمال وجود دارد
که می توانند از ان برای ارسال پیام های کلاسیک استفاده کنند.

Definition: Entanglement-assisted Classical Capacity of Quantum Channels



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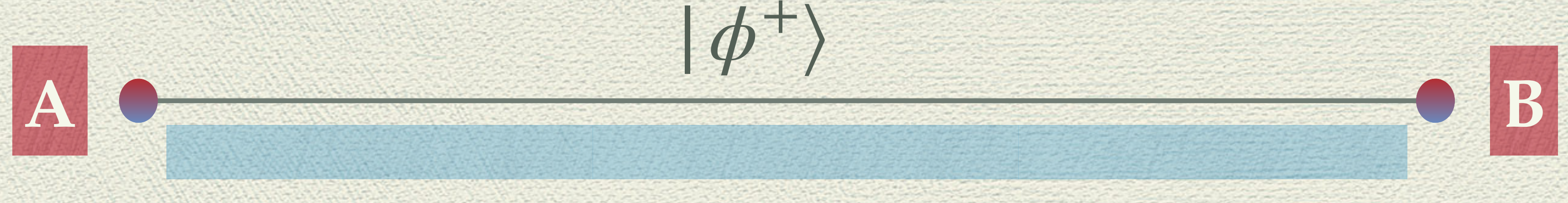
Example: Dense Coding

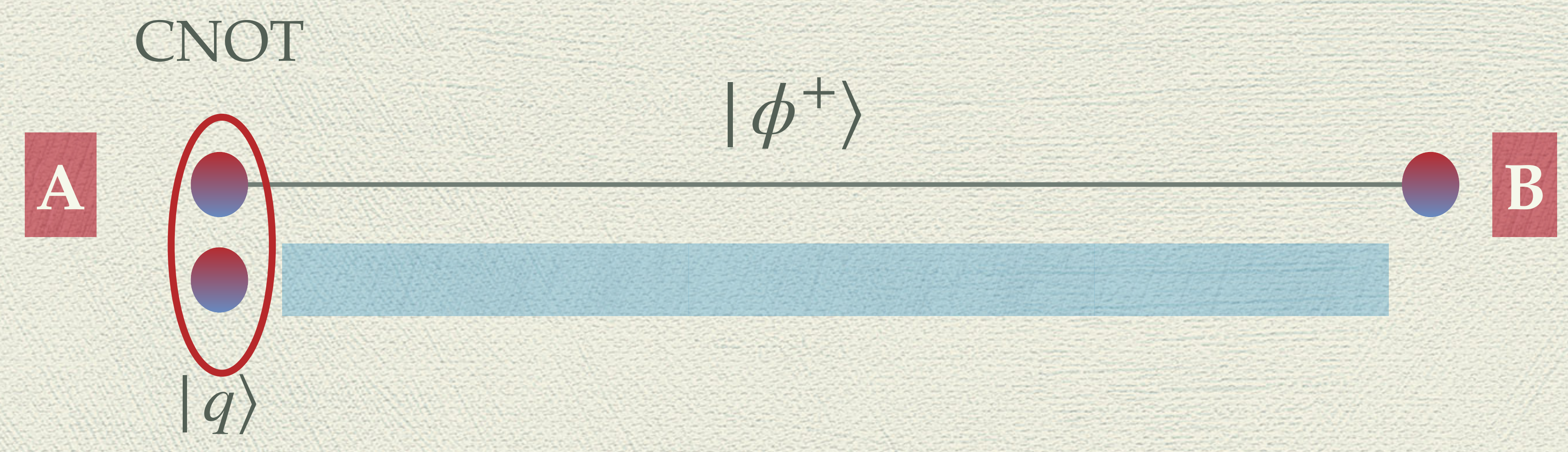
Example 1: Dense Coding



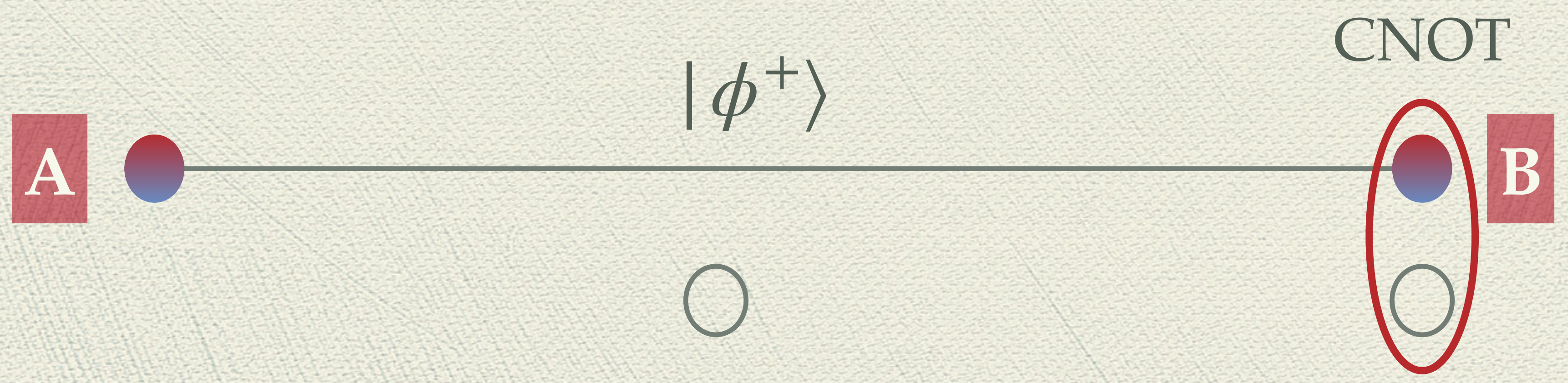
$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

I	$\lvert\phi^+\rangle = \frac{1}{\sqrt{2}}(\lvert 00\rangle + \lvert 11\rangle)$	00
X	$\lvert\phi^+\rangle = \frac{1}{\sqrt{2}}(\lvert 10\rangle + \lvert 01\rangle)$	01
Y	$\lvert\phi^+\rangle = \frac{1}{\sqrt{2}}(\lvert 10\rangle - \lvert 01\rangle)$	10
Z	$\lvert\phi^+\rangle = \frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 11\rangle)$	11

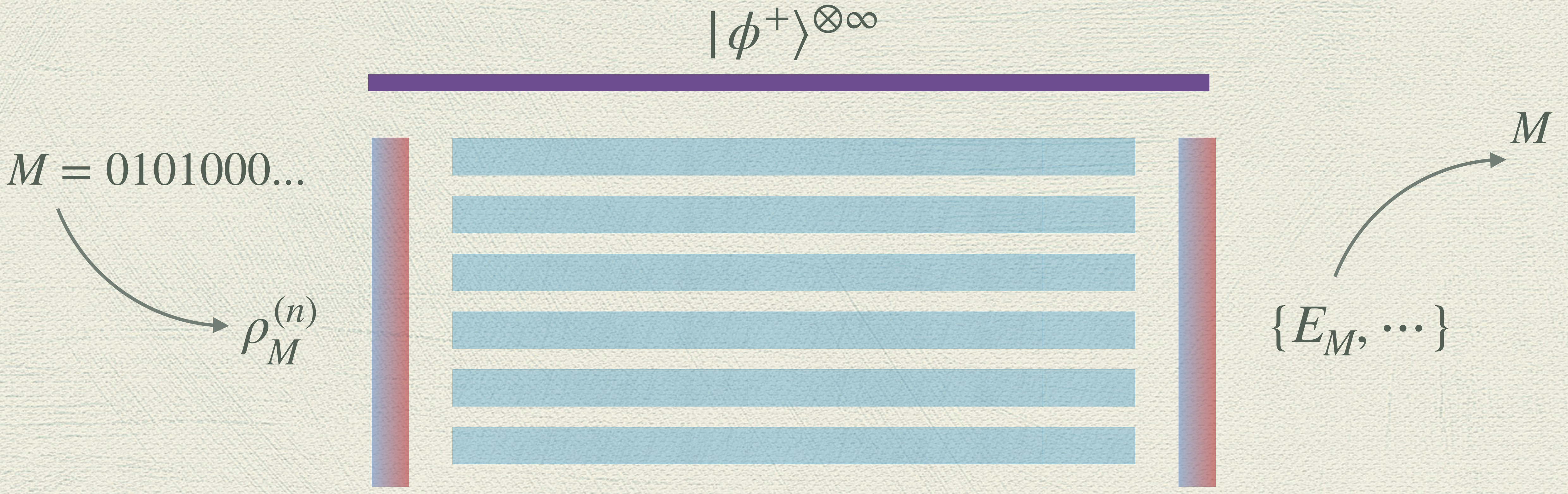
Example 2



$$|\Psi\rangle_{ABA} = \frac{1}{\sqrt{2}}(|00q\rangle + |11\bar{q}\rangle)$$



<u>General Definition</u>: Entanglement-assisted Classical Capacity of Quantum Channels



$\rho_M^{(n)}$ =Quantum state of n-qubits

$$C = Max_{n\longrightarrow\infty}\frac{|M|}{n} \quad ?$$

Calculation: Entanglement-Assisted Classical Capacity of Quantum Channel

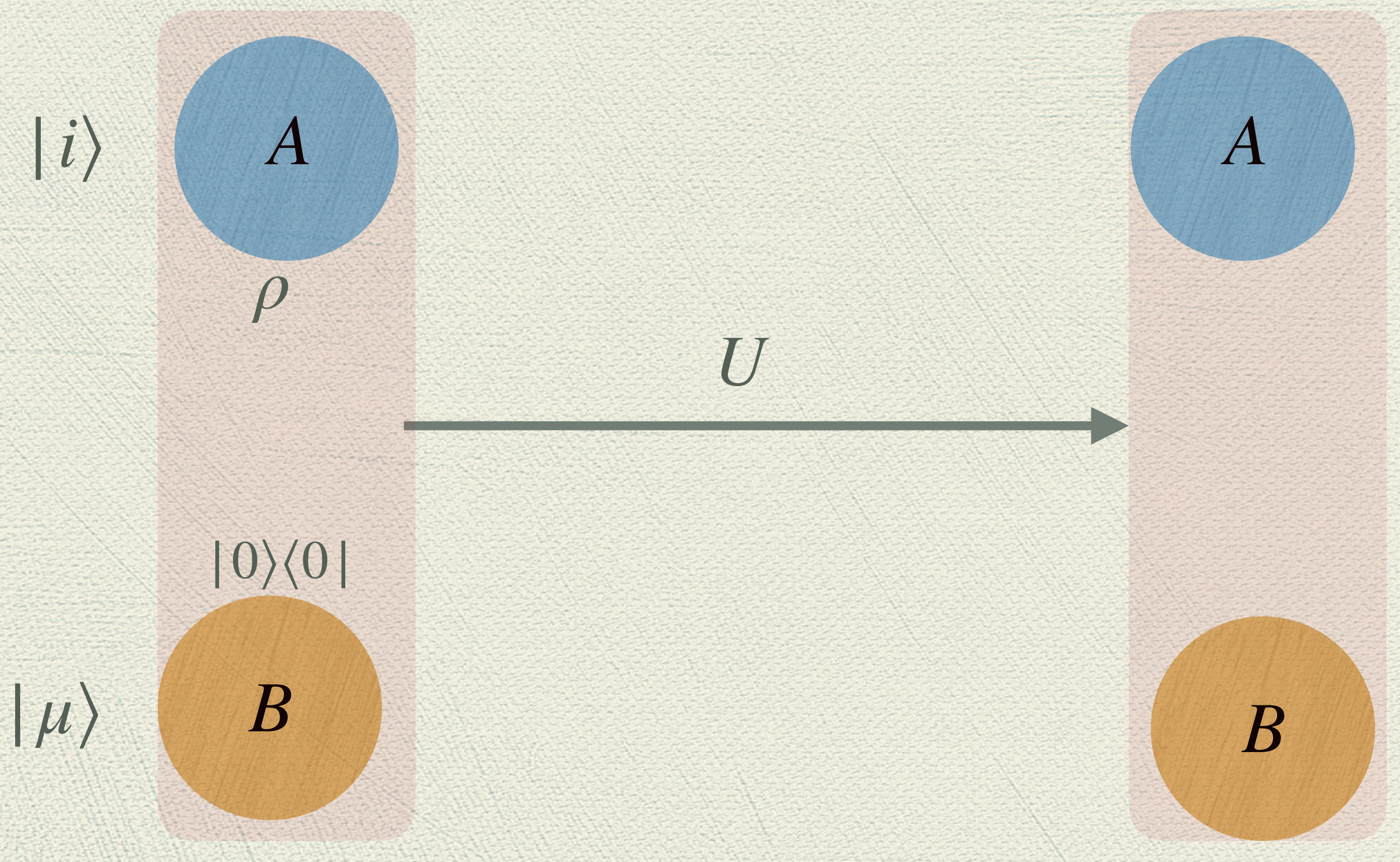
$$C_E(\Phi) = \underset{\rho}{Max}\; I(\rho, \Phi)$$

$$I(\rho, \Phi) = S(\rho) + S(\Phi(\rho)) - S(\Phi^c(\rho))$$

بنابراین برای این نوع ظرفیت یک فرمول بسته و ساده وجود دارد.

ولی بازهم می بایست این عبارت آخری را روی تمام ماتریس های چگالی بیشینه کرد.

Complementary Channels

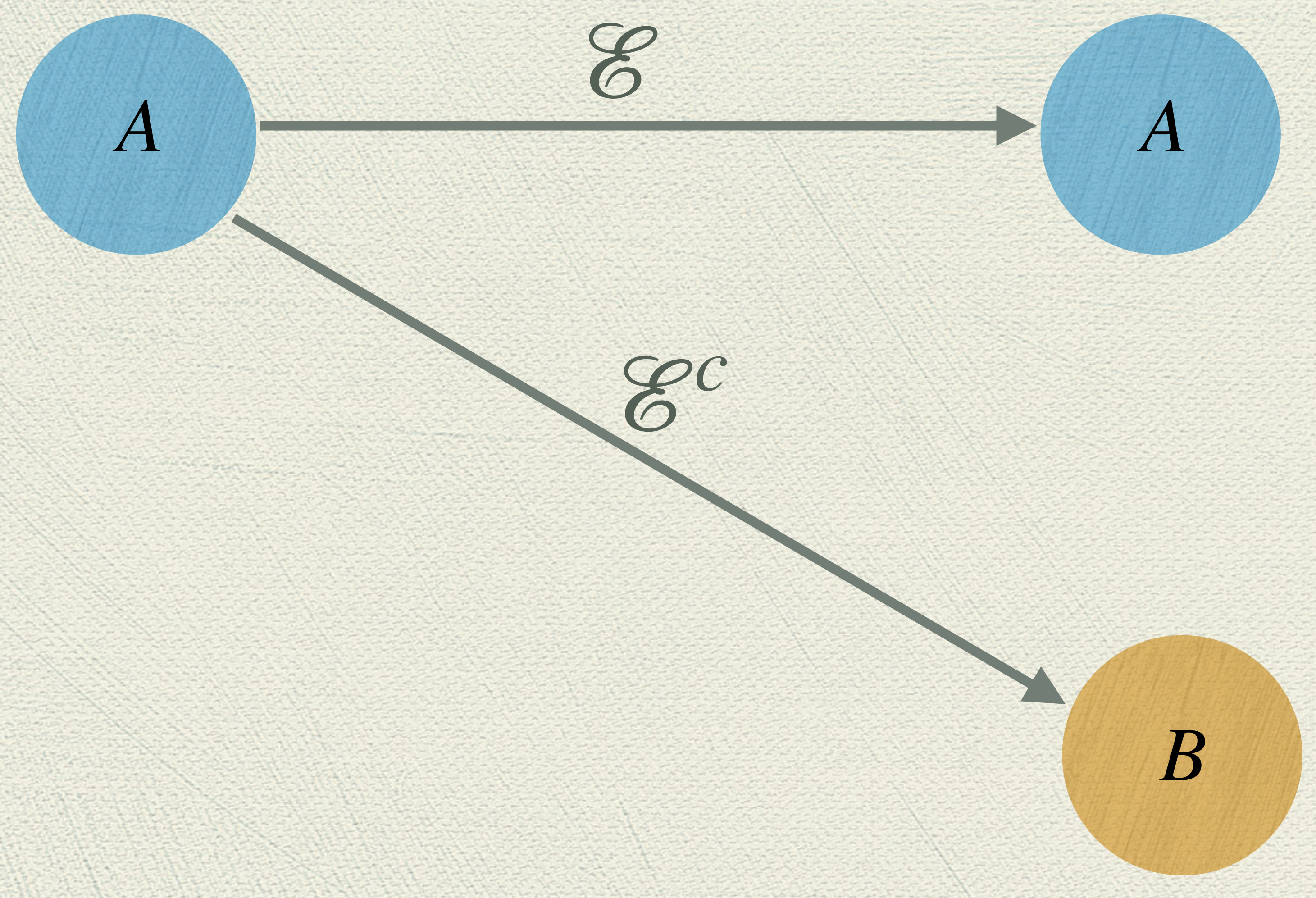


Complementary Channels

$$\mathscr{E}(\rho) = Tr_B\Big[U(\rho \otimes |0\rangle\langle 0|)U^\dagger\Big] \qquad \in A$$

$$\mathscr{E}^c(\rho) = Tr_A\Big[U(\rho \otimes |0\rangle\langle 0|)U^\dagger\Big] \qquad \in B$$

Complementary Channels



Kraus Decomposition

$$\mathscr{E}(\rho) = \sum_{\mu} A_\mu \rho A_\mu{}^\dagger$$

$$(K_i)_{\mu,j} = (A_\mu)_{ij}$$

$$\mathscr{E}^c(\rho) = \sum_i K_i \rho K_i^\dagger$$

ماتریس K_1 را از سطرهای اول A ها می سازیم.

ماتریس K_2 را از سطرهای دوم A ها می سازیم.

ماتریس K_3 را از سطرهای سوم A ها می سازیم.


$\mathcal{E}$
A_1
A_2
A_3
$\mathcal{E}^c$
K_1
K_2
K_3
K_4

Example 1: Complement of Bit-flip Channel

$$\mathcal{E}(\rho) = (1-p)\rho + p\sigma_x\rho\sigma_x$$

$$A_0 = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & \sqrt{p} \\ \sqrt{p} & 0 \end{pmatrix}$$

$$K_0 = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{p} \end{pmatrix} \qquad K_1 = \begin{pmatrix} 0 & \sqrt{1-p} \\ \sqrt{p} & 0 \end{pmatrix}$$

$$\rho = \frac{1}{2}(I + \mathbf{r} \cdot \sigma)$$

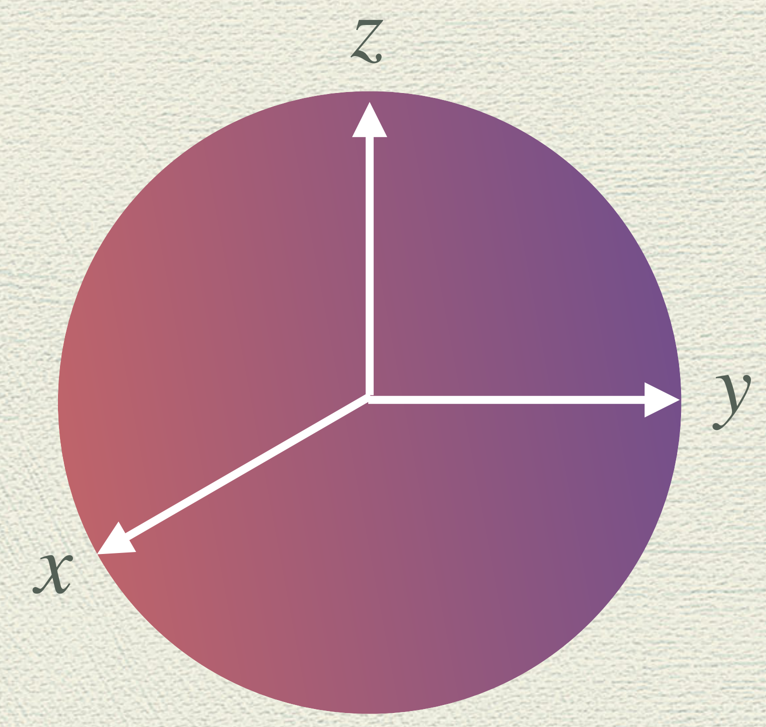
$$\mathbf{r} = (x, y, z)$$

$$\mathcal{E}(\rho) = \frac{1}{2}(I + \mathbf{r}' \cdot \sigma)$$

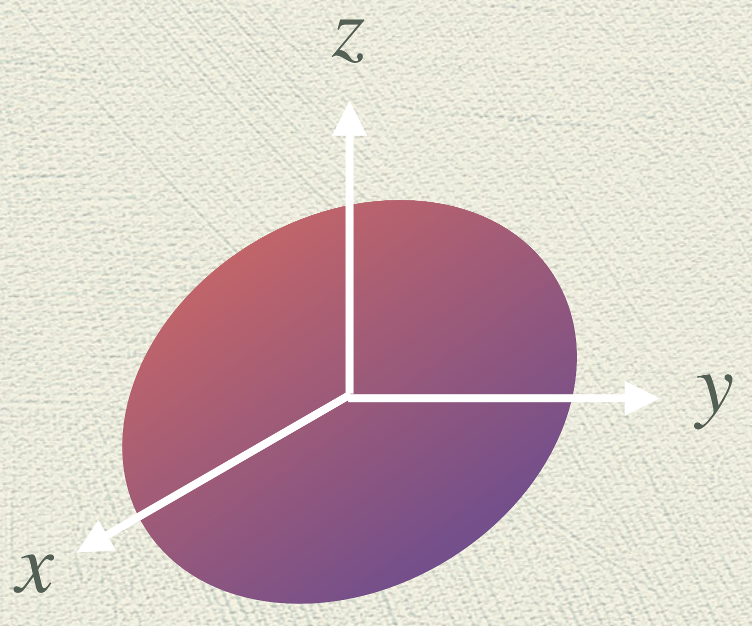
$$\mathbf{r}' = (x, (1-2p)y, (1-2p)z)$$

$$\mathcal{E}^c(\rho) = \frac{1}{2}(I + \mathbf{r_c}' \cdot \sigma)$$

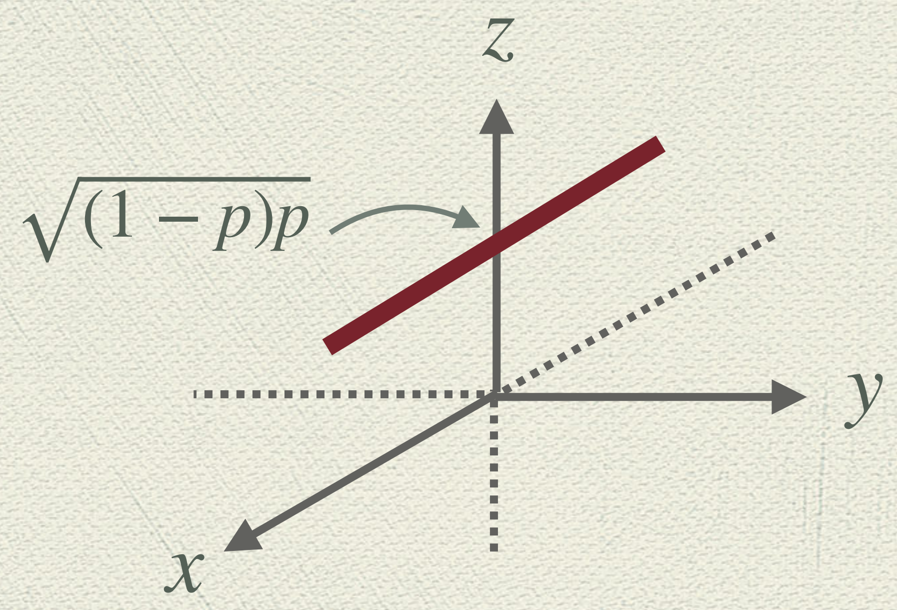
$$\mathbf{r_c}' = (\sqrt{(1-p)p}\ x,\ 0,\ (1-2p))$$



ρ



$\mathscr{E}(\rho)$



$\mathscr{E}^c(\rho)$

$$I(\rho, \Phi) = S(\rho) + S(\Phi(\rho)) - S(\Phi^c(\rho))$$

Example 2: Depolarizing Channel

$$\mathscr{E}(\rho) = (1 - \frac{3p}{4})\rho + \frac{p}{4}(\sigma_x\rho\sigma_x + \sigma_y\rho\sigma_y + \sigma_z\rho\sigma_z)$$

$$A_0 = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A_1 = \beta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A_2 = \beta \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad A_3 = \beta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\alpha = \sqrt{1 - \frac{3p}{4}} \qquad \beta = \sqrt{\frac{p}{4}}$$

$$A_0 = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A_1 = \beta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A_2 = \beta \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad A_3 = \beta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \\ 0 & -i\beta \\ \beta & 0 \end{pmatrix} \qquad R_2 = \begin{pmatrix} 0 & \alpha \\ \beta & 0 \\ i\beta & 0 \\ 0 & -\beta \end{pmatrix}$$

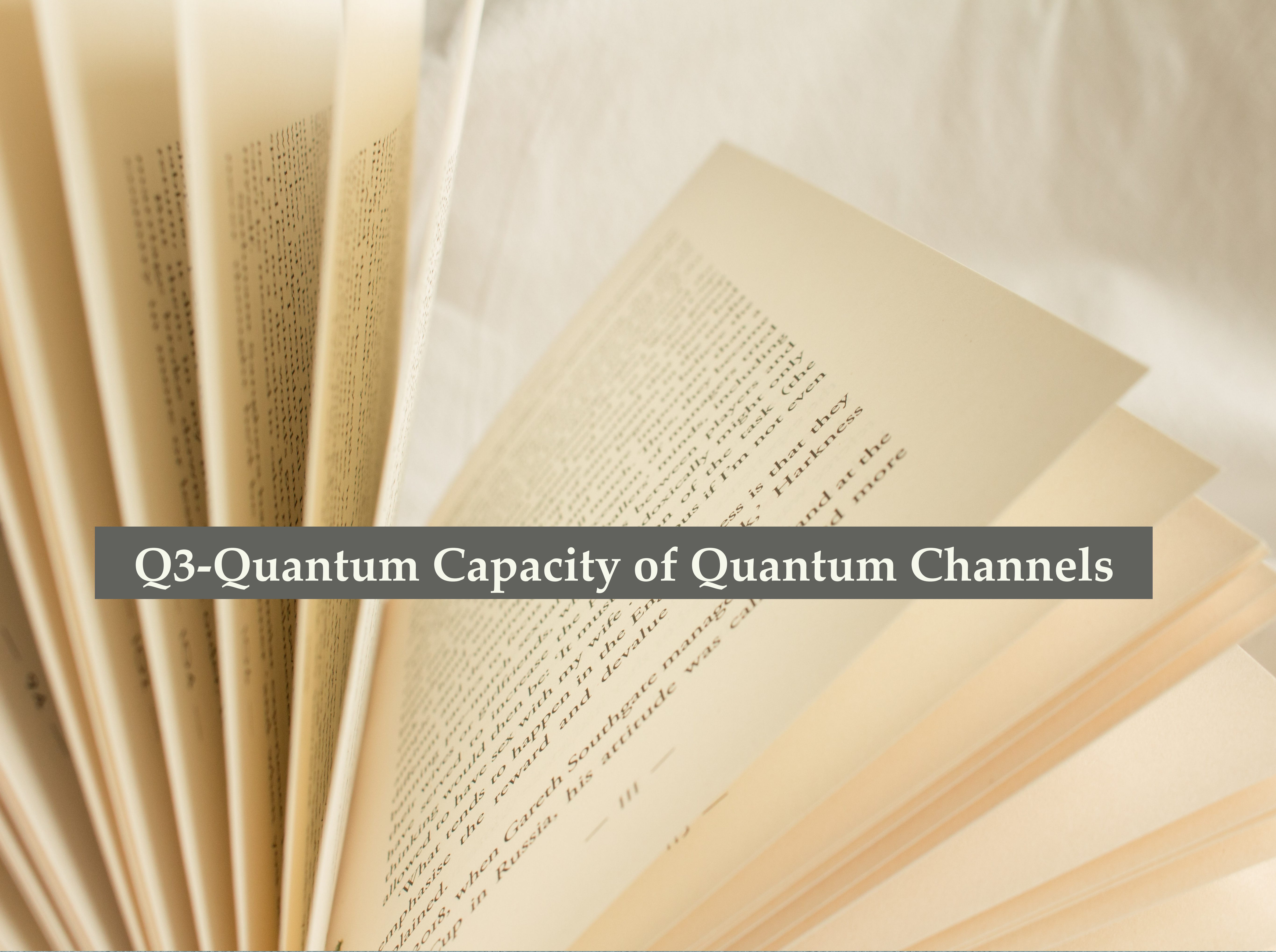
$$\mathcal{E}^c(\rho) = \begin{pmatrix} \alpha^2 & \alpha\beta x & \alpha\beta y & \alpha\beta z \\ \alpha\beta x & \beta & -i\beta z & i\beta y \\ \alpha\beta y & i\beta z & \beta & -i\beta x \\ \alpha\beta z & -i\beta y & i\beta x & \beta \end{pmatrix}$$

Eigenvalues of $\rho : \{\frac{1}{2}(1 \pm r)\}$

Eigenvalues of $\mathscr{E}(\rho) : \{\frac{1}{2}(1 \pm (1-p)r)\}$

Eigenvalues of $\mathscr{E}^c(\rho) : \quad \{\beta^2(1 \pm r), \frac{1}{2}(1 - 2\beta^2 \pm \sqrt{(1-4\beta^2)^2 - 8(\beta^2 r^2)^2})\}$

$$I(\rho, \Phi) = S(\rho) + S(\Phi(\rho)) - S(\Phi^c(\rho))$$



Q3-Quantum Capacity of Quantum Channels

Definition: Quantum Capacity of Quantum Channels

Quantum Channel Φ

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

ρ

Definition: Quantum Capacity of Quantum Channels

Definition: Quantum Capacity of Quantum Channels

We want $\overline{F}(|\psi_k'\rangle, |\psi_k\rangle) \geq 1 - \epsilon$

$$R(X) = \lim_{n \longrightarrow \infty} \frac{k}{n}$$

Definition: Quantum Capacity of Quantum Channels

$$C = \underset{X}{Max}\ R(X)$$

Calculation: Quantum Capacity of Quantum Channels

$$J_n(\rho, \Phi) = S(\Phi^{\otimes n}(\rho)) - S((\Phi^c)^{\otimes n}(\rho))$$

$$J_n(\Phi) = \underset{\rho}{Max}\ J_n(\rho, \Phi^{\otimes n})$$

$$C = \lim_{n \longrightarrow \infty} \frac{1}{n} J_n[\Phi]$$

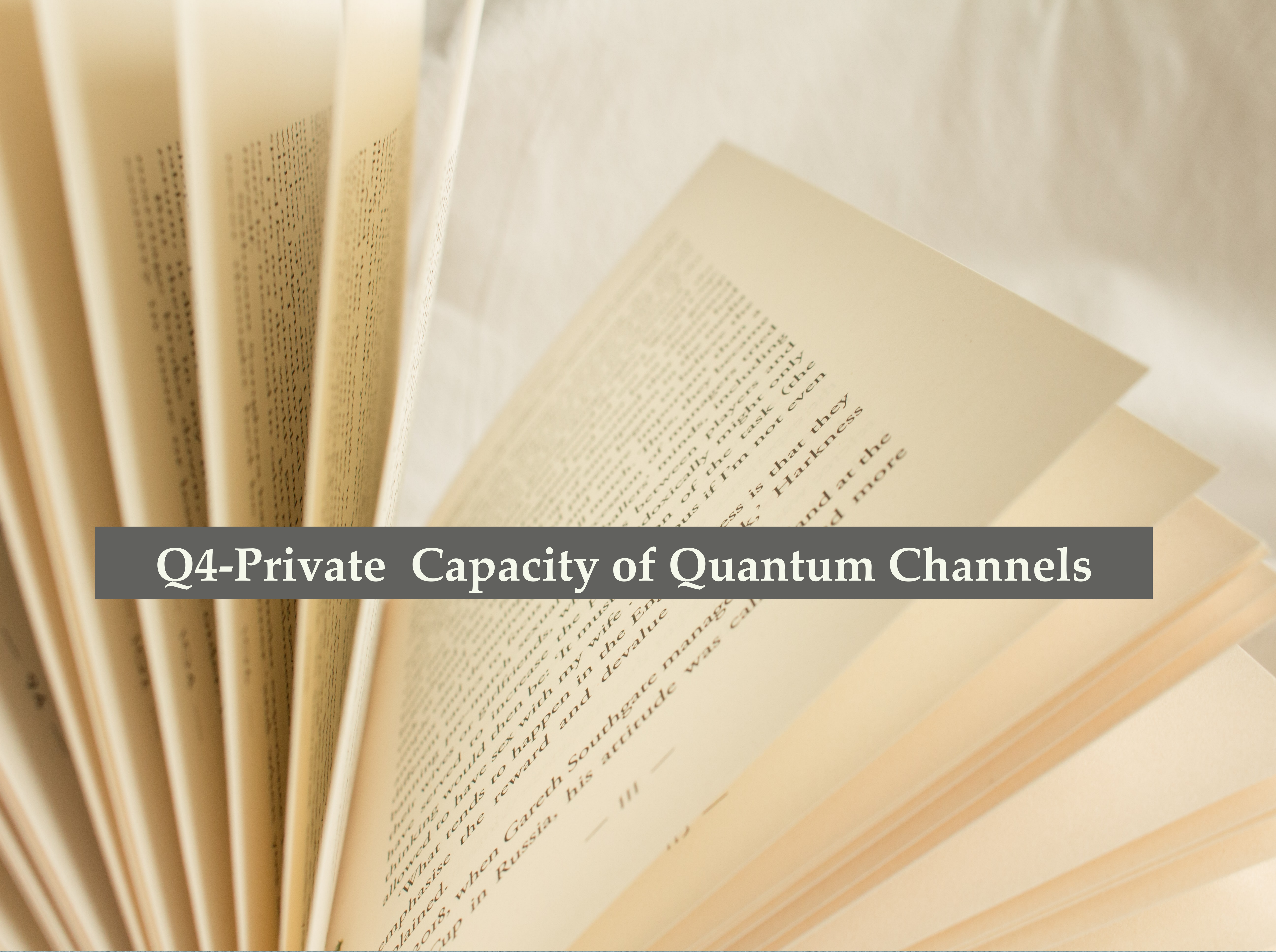
Obviously this capacity cannot be calculated,

Except when the channel is degradable.

For degradable channels

$$C = J[\Phi] = \underset{\rho}{Max} \quad S[\Phi(\rho)] - S[\Phi^c(\rho)]$$

Q4-Private Capacity of Quantum Channels



$$Pr(M \neq M') \leq \epsilon$$

$$\|\rho_{BE} - \rho_B \otimes \rho_E\| \leq \delta$$

Transferring classical messages to Bob without any leakage to Eve

Single Shot Capacity $$C_p^1(\Lambda) = max_{\{p_i,\rho_i\}}\left[\chi(\Lambda) - \chi(\Lambda^c)\right]$$

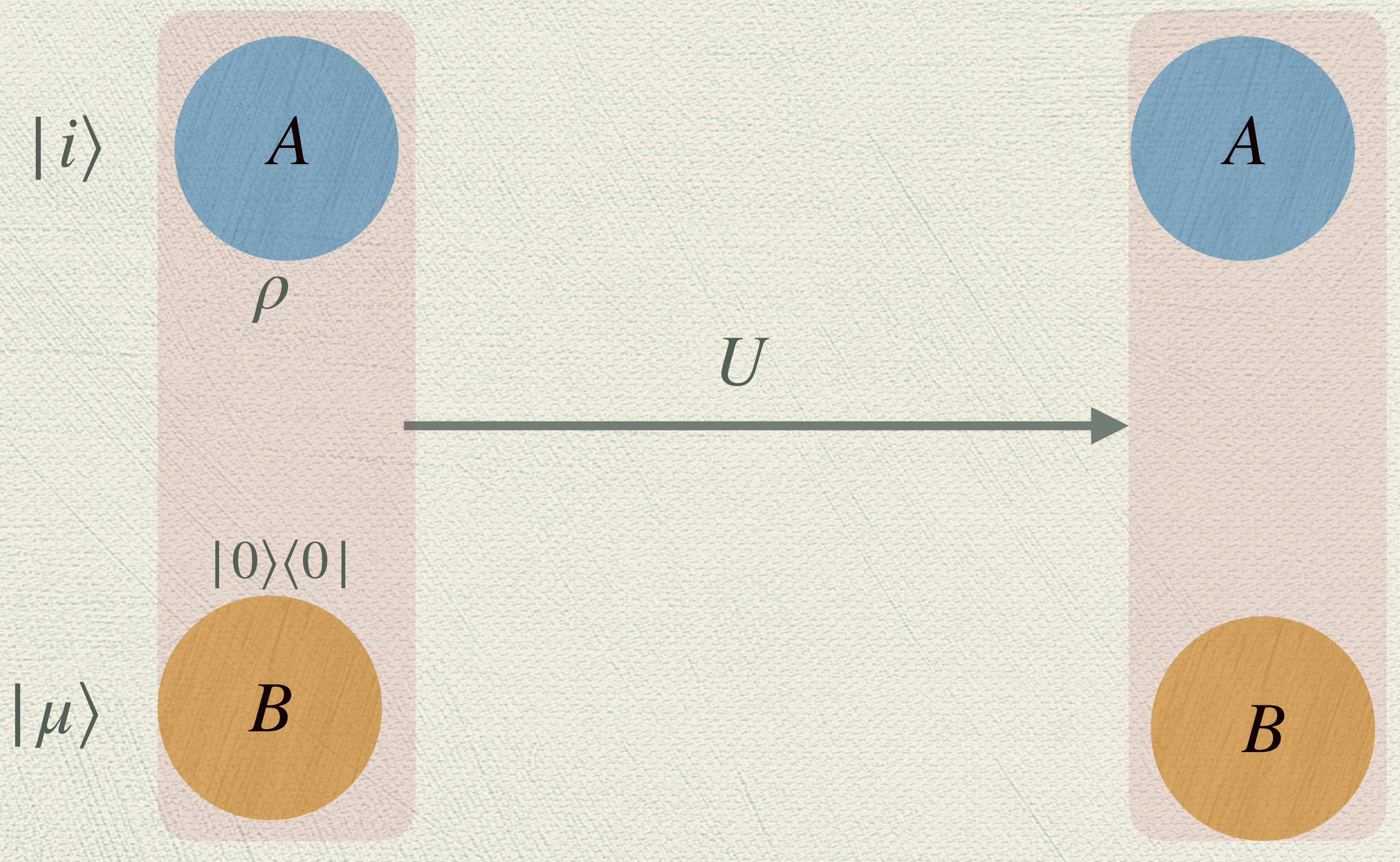
$$C_p(\Lambda) = Max_{\{p_i,\rho_i\}} \lim_{n\longrightarrow\infty} \left[\chi(\Lambda^{\otimes n}) - \chi(\Lambda^{c^{\otimes n}})\right]$$

$$C_p^1(\Lambda) \leq C_p(\Lambda)$$

$$Q(\Lambda) \leq C_p(\Lambda)$$

Degradable Channels

Complementary Channels

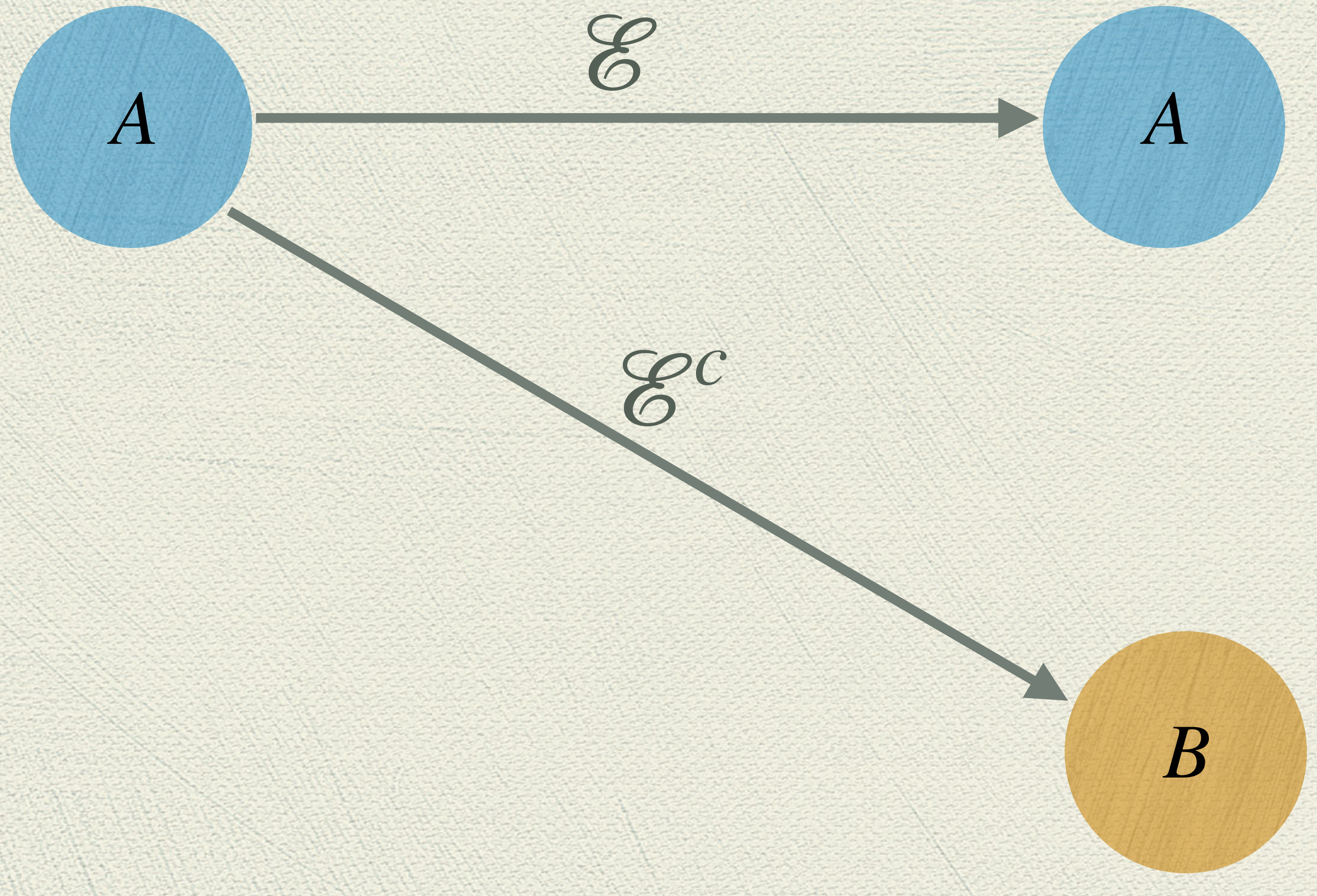


Complementary Channels

$$\mathcal{E}(\rho) = Tr_B\Big[U(\rho \otimes |0\rangle\langle 0|)U^\dagger\Big] \qquad \in A$$

$$\mathcal{E}^c(\rho) = Tr_A\Big[U(\rho \otimes |0\rangle\langle 0|)U^\dagger\Big] \qquad \in B$$

Complementary Channels



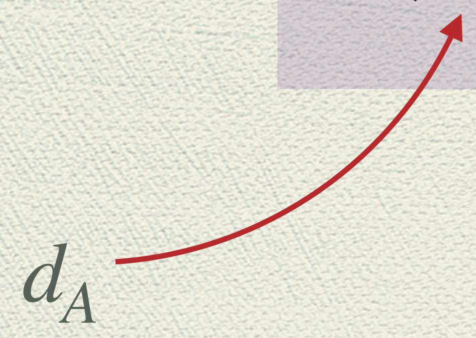
Kraus Decomposition

$$\mathcal{E}(\rho) = \sum_{\mu} A_\mu \rho A_\mu^\dagger \qquad (A_\mu)_{ij} = \langle i, \mu | U | j, 0 \rangle$$

$$\mathcal{E}^c(\rho) = \sum_{i} K_i \rho K_i^\dagger \qquad (K_i)_{\mu,j} = \langle i, \mu | U | j, 0 \rangle$$

$$(K_i)_{\mu,j} = (A_\mu)_{ij}$$

$$(K_i)_{\mu,j} = (A_\mu)_{ij}$$



Example 1: Complement of Bit-flip Channel

$$(K_i)_{\mu,j} = (A_\mu)_{i,j}$$

$$\mathcal{E}(\rho) = (1-p)\rho + p\sigma_x\rho\sigma_x$$

$$A_0 = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & \sqrt{p} \\ \sqrt{p} & 0 \end{pmatrix}$$

$$K_0 = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{p} \end{pmatrix} \qquad K_1 = \begin{pmatrix} 0 & \sqrt{1-p} \\ \sqrt{p} & 0 \end{pmatrix}$$

Example 2: Complement of Pauli Channel

$$\mathscr{E}(\rho) = p_0\rho + p_1\sigma_x\rho\sigma_x + p_2\sigma_y\rho\sigma_y + p_3\sigma_z\rho\sigma_z$$

$$K_0 = \begin{pmatrix} \sqrt{p_0} & 0 \\ 0 & \sqrt{p_1} \\ 0 & -i\sqrt{p_2} \\ \sqrt{p_3} & 0 \end{pmatrix} \qquad K_1 = \begin{pmatrix} 0 & \sqrt{p_0} \\ \sqrt{p_1} & 0 \\ i\sqrt{p_2} & 0 \\ 0 & -\sqrt{p_3} \end{pmatrix}$$

More Examples

$$\mathcal{E}_0(\rho) = Tr(\rho)\frac{I}{d}$$

$$\tilde{\mathcal{E}}_0(\rho) = \frac{I}{d}(\rho \otimes I)$$

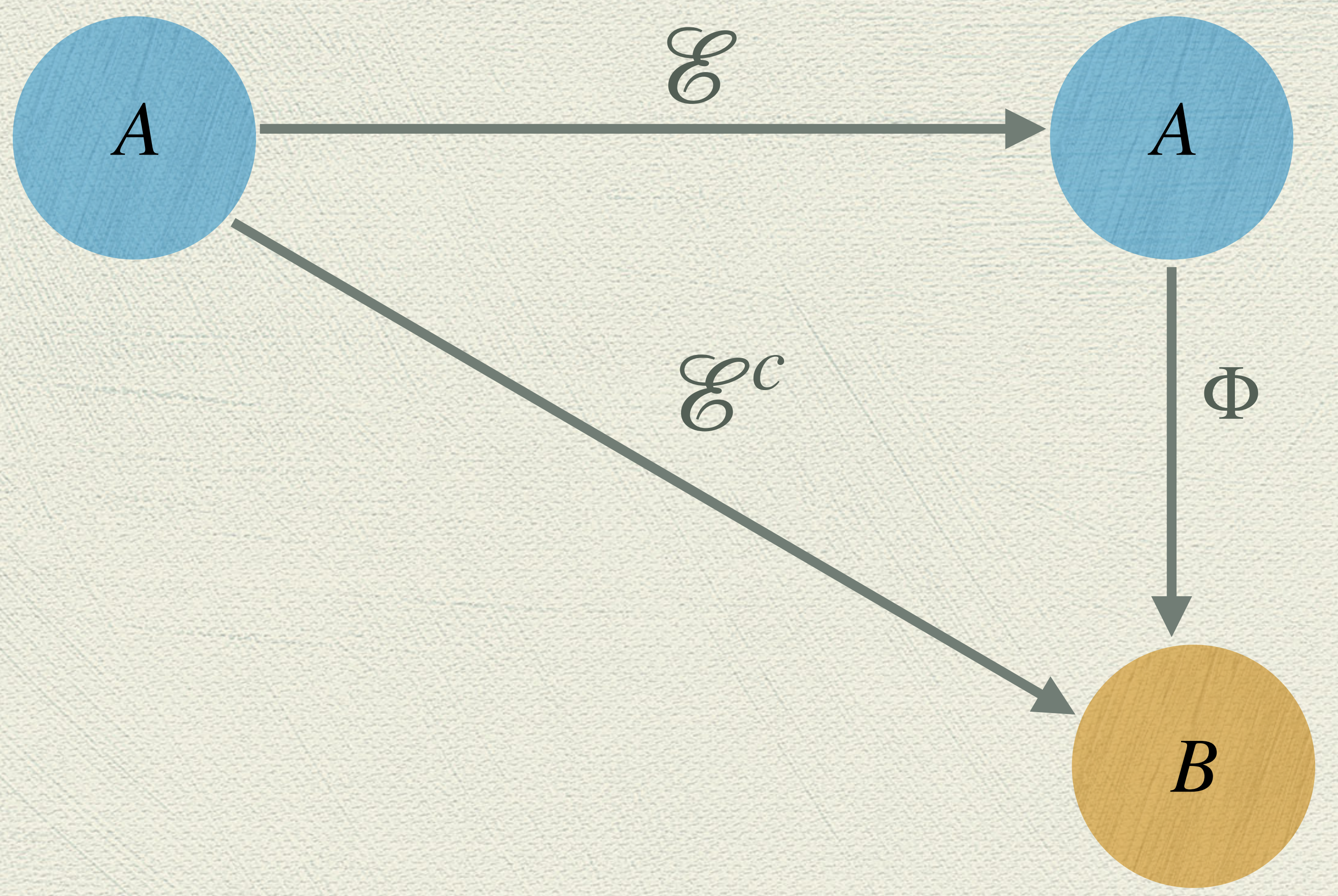
$$\mathcal{E}_U(\rho) = U\rho U^\dagger$$

$$\tilde{\mathcal{E}}_U(\rho) = Tr(\rho)$$

$$\mathcal{E}_\psi(\rho) = Tr(\rho)\,|\psi\rangle\langle\psi|$$

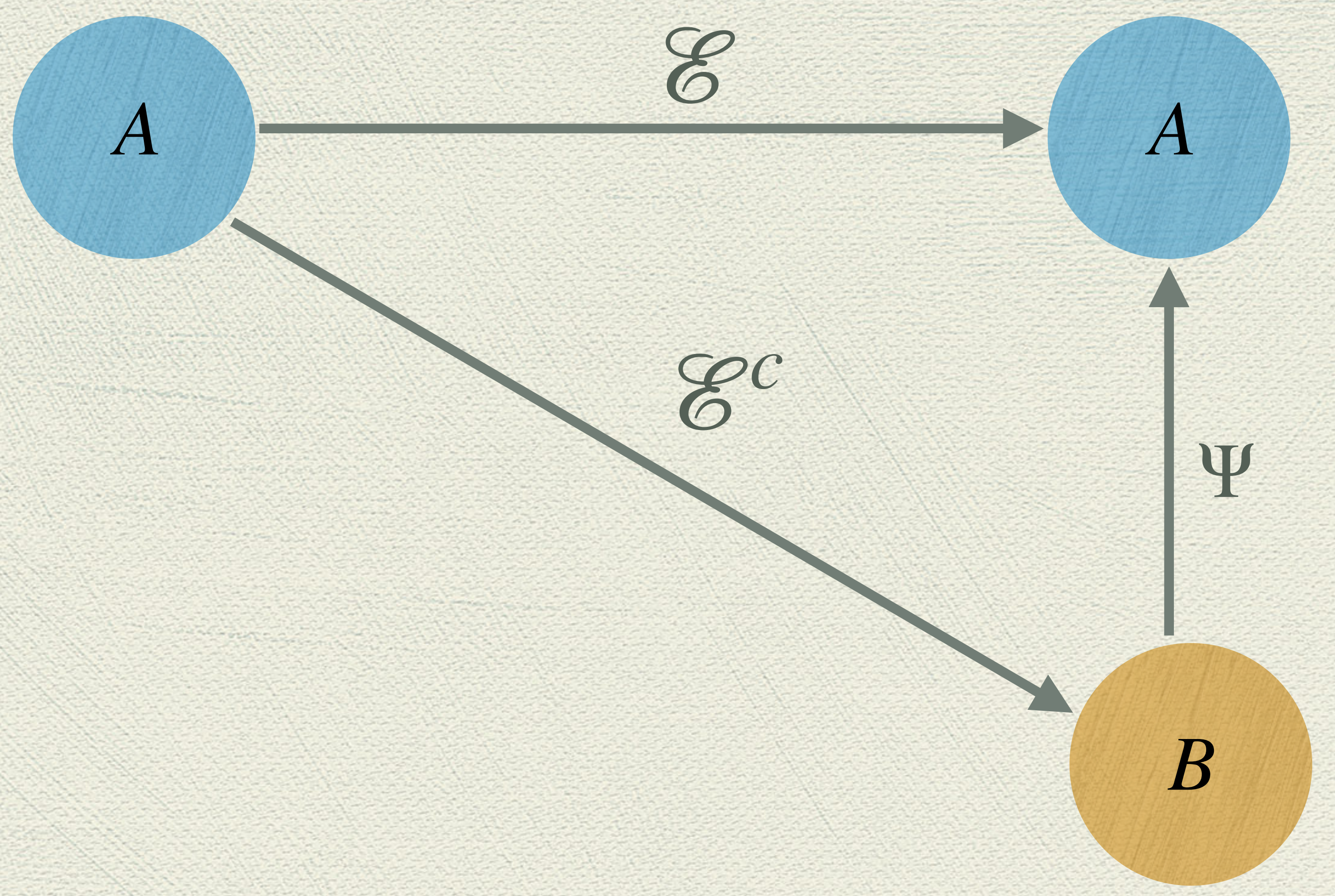
$$\mathcal{E}^c_\psi(\rho) = \rho$$

Degradable Channels



$$\mathscr{E}^c = \Phi \circ \mathscr{E}$$

Anti-Degradable Channels



$$\mathscr{E} = \Psi \circ \mathscr{E}^c$$

Examples of degradable and anti-degradable Channels

$$\mathcal{E}_0(\rho) = Tr(\rho)\frac{I}{d} \qquad \tilde{\mathcal{E}}_0(\rho) = \frac{I}{d}(\rho \otimes I)$$

$$\Phi \circ \tilde{\mathcal{E}}_0 = \mathcal{E}_0$$

$$\Phi(X) = Tr_A(X)$$

Anti-Degradable

Examples of degradable and anti-degradable Channels

$$\mathscr{E}_U(\rho) = U\rho U^\dagger \qquad \tilde{\mathscr{E}}_U(\rho) = Tr(\rho)$$

$$\tilde{\mathscr{E}}_U = \Phi \circ \mathscr{E}_U$$

$$\Phi(X) = Tr(X)$$

Degradable

Examples of degradable and anti-degradable Channels

$$\mathcal{E}_\psi(\rho) = Tr(\rho)\,|\psi\rangle\langle\psi| \qquad \tilde{\mathcal{E}}_\psi(\rho) = \rho$$

$$\mathcal{E}_\psi = \Phi \circ \tilde{\mathcal{E}}_\psi$$

$$\Phi(X) = Tr(X)\,|\psi\rangle\langle\psi|$$

Anti-Degradable

Is there any non-trivial example of a degradable channel?

For which we can calculate the quantum capacity?

Q4: Private Capacity of Quantum Channels

For more information and for details, see:

Capacities of the covariant Pauli channel
A Poshtvan, V Karimipour
Physical Review A 106 (6), 062408

End of part III